

# Factorized power expansion for high- $p_T$ heavy-quarkonium production

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Based on work done with Zhong-Bo Kang, Yan-Qing Ma,  
Jian-Wei Qiu, and George Sterman

PRD 89.094029/094030, PRD 90.034004, PRL 113.142002, PRD 91.014030, arXiv: 1501.04556

Charm 2015  
Wayne State University, 2015/05/19

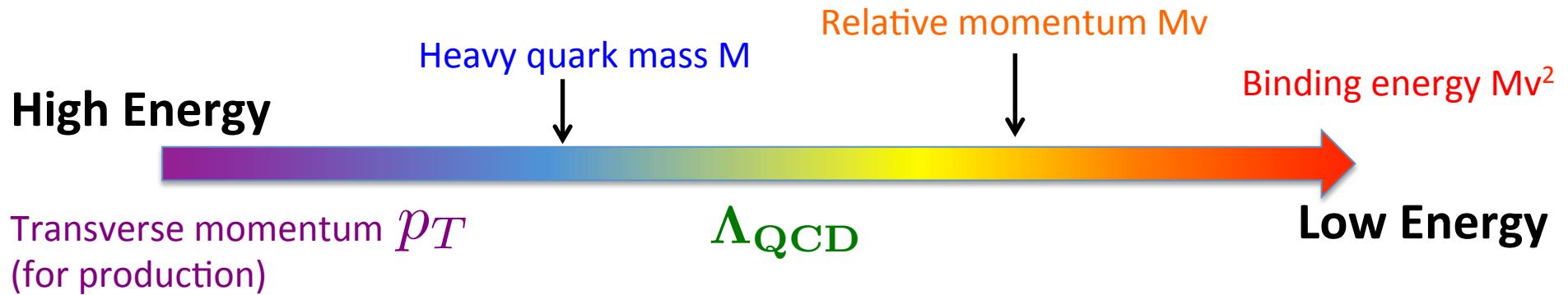
# Heavy quarkonium & models

- ❖ Non-relativistic QCD system

Charmonium:  $v^2 \approx 0.3$

Bottomonium:  $v^2 \approx 0.1$

- ❖ Multiple separated scales



- ❖ Historical models for production

- Color Singlet Model (CSM): 1975
- Color Evaporation Model (CEM): 1977
- NRQCD factorization: 1986, 1994

Einhorn and Ellis (1975), Chang (1980),  
Berger and Jones (1981)...

Fritsch (1977), Halzen (1977), ...

Caswell, Lapage (1986)  
Bodwin, Braaten, Lepage (1995)

# NRQCD Factorization Formalism

- ❖ NRQCD factorization for HQ production

Bodwin, Braaten, Lepage (1995)

See Bodwin's talk

$$d\sigma_{AB \rightarrow \psi + X} = \sum_{c\bar{c}[n]} d\hat{\sigma}_{AB \rightarrow c\bar{c}[n] + X} \times \langle \mathcal{O}_n^\psi \rangle$$

Double expansion in  $\alpha_s$  and  $v$

$$n : {}^{2S+1}L_J^{[1,8]}$$

- ❖ NRQCD long-distance matrix elements (LDMEs)

**Nonperturbative  
but universal**

$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[1]}) \rangle \sim v^3$$

CSM channel  
can be obtained from decay data

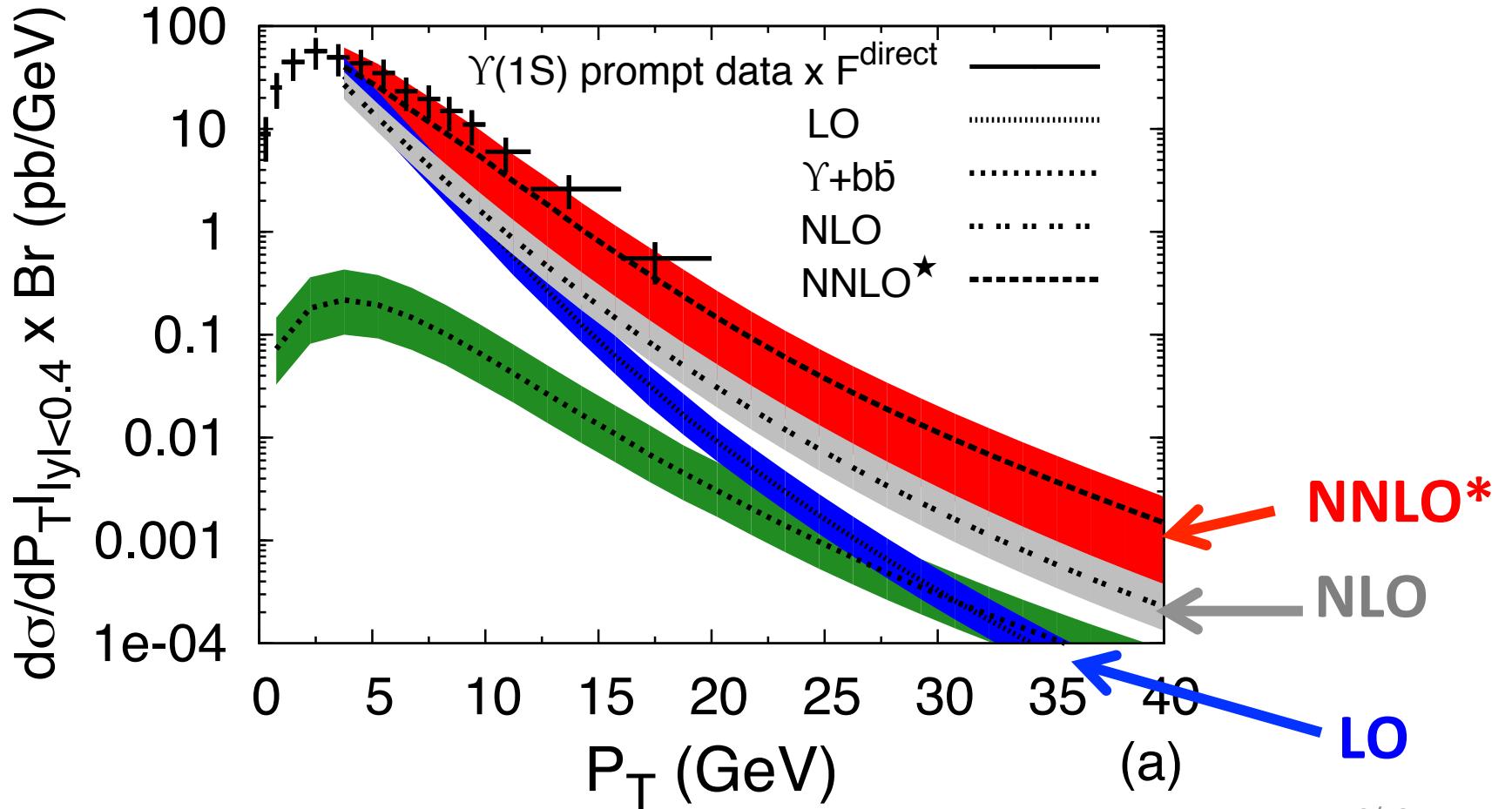
$$\langle \mathcal{O}^{J/\psi}(^3S_1^{[8]}) \rangle, \langle \mathcal{O}^{J/\psi}(^1S_0^{[8]}) \rangle, \langle \mathcal{O}^{J/\psi}(^3P_J^{[8]}) \rangle \sim v^7$$

Color-octet channels, need to be fitted from production data

# Large high-order correction

- ❖ High-order corrections are orders of magnitude larger!

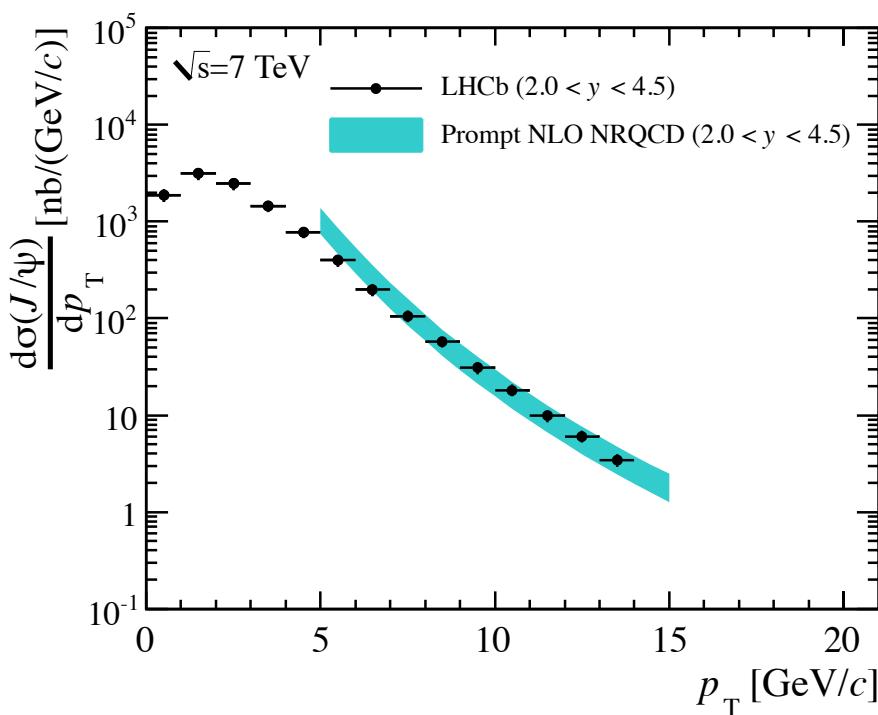
$^3S_1^{[1]}$  channel



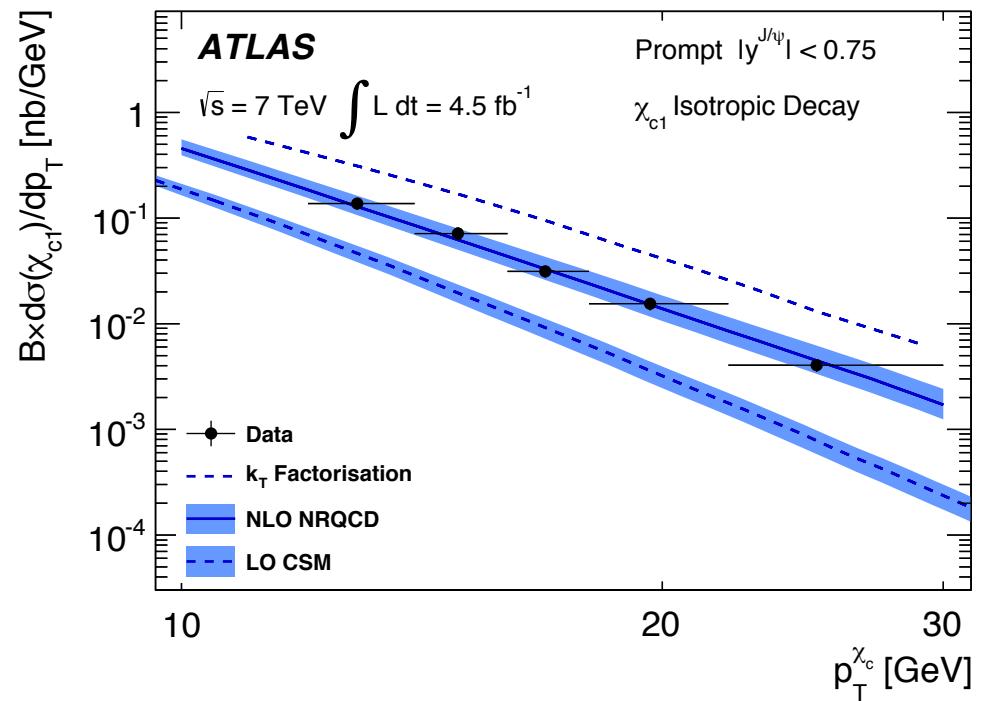
Campbell et al. PRL 2007. Artoisenet et al. PRL 2008

# Inclusive Polarization-summed J/psi

- ❖ NLO NRQCD calculation explains data for a large pT range

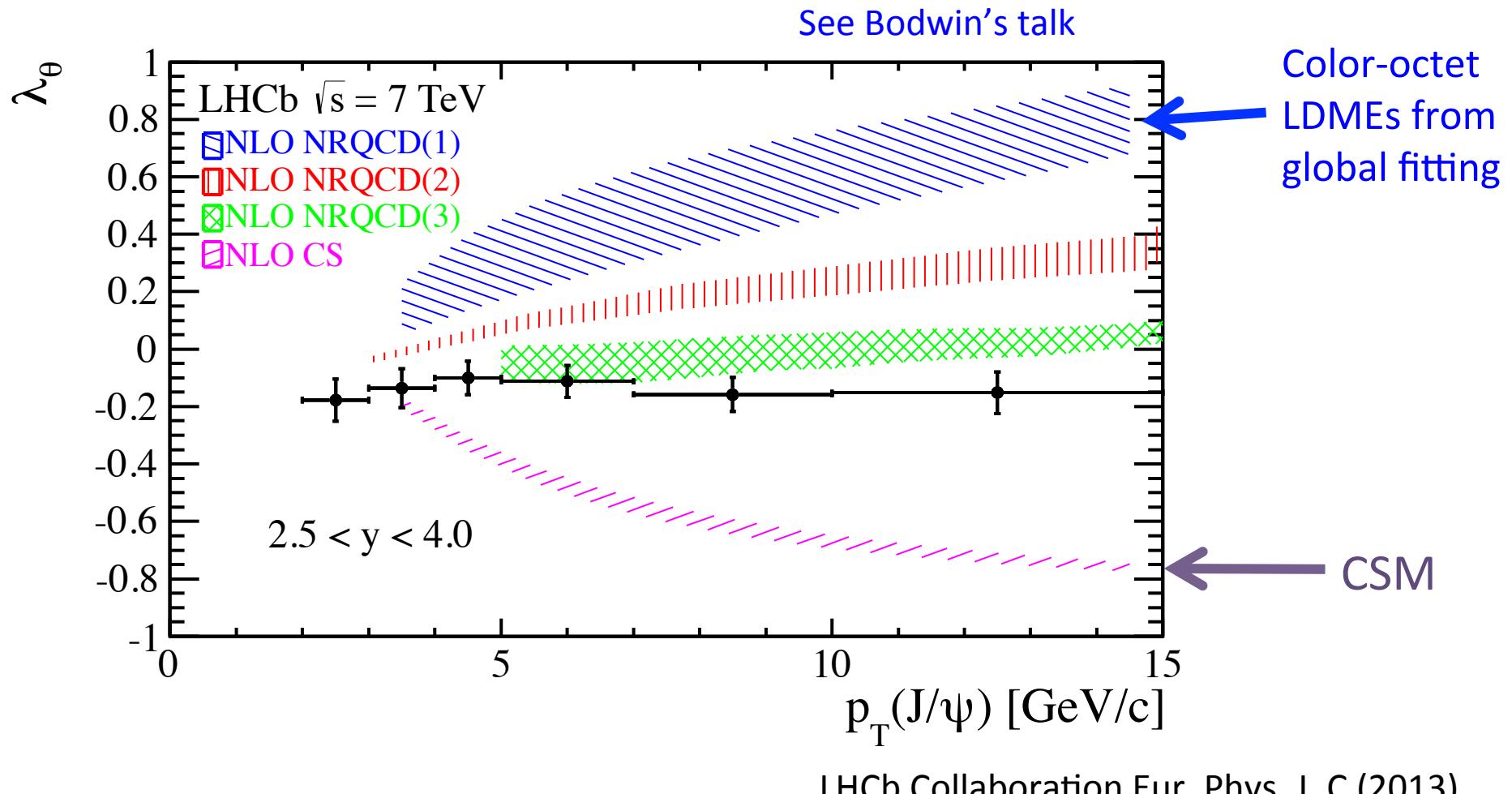


LHCb collaboration, 2011



Atlas collaboration, 2014

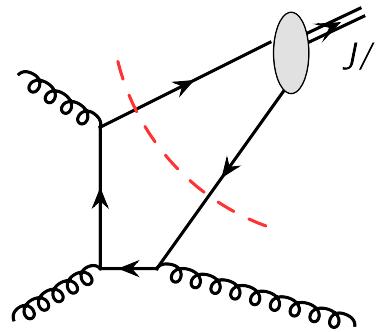
# Puzzles – Polarized J/psi production



The three NRQCD predictions differ only by the color-octet NRQCD LDMEs

# Large High-order Correction: Revisit

$^3S_1^{[1]}$  channel

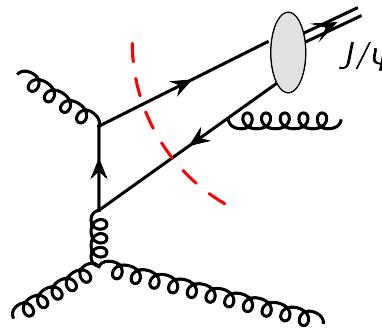


- LO in  $\alpha_s$   
 $\text{NNLP} \propto \alpha_s^3 \frac{m_Q^4}{p_T^8}$

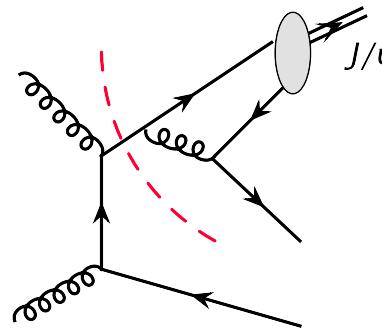
Effectively

$$\frac{d\hat{\sigma}}{dp_T^2} \approx \alpha_s^3 \frac{m_Q^4}{p_T^8} C^{(0)} + \alpha_s^4 \frac{m_Q^2}{p_T^6} \log\left(\frac{m_Q^2}{p_T^2}\right) C^{(1)} + \alpha_s^5 \frac{1}{p_T^4} \log^n\left(\frac{m_Q^2}{p_T^2}\right) C^{(2)} + \mathcal{O}(\alpha_s^6)$$

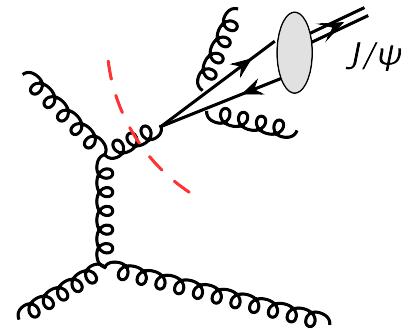
See Qiu's talk



- NLO in  $\alpha_s$   
dominated by NLP,  $\propto \alpha_s^4 \frac{m_Q^2}{p_T^6}$



- NNLO in  $\alpha_s$   
dominated by LP,  
 $\propto \alpha_s^5 \frac{1}{p_T^4}$



- Expect no further power enhancement beyond NNLO
- $\alpha_s \ln(m_Q^2/p_T^2)$  ruins the perturbation series at sufficiently large  $p_T$

# NLP QCD Factorization

## ❖ Factorization formalism

$$d\sigma_{A+B \rightarrow H+X}(p_T) = \sum_f d\hat{\sigma}_{A+B \rightarrow f+X}(p_f = p/z) \otimes D_{H/f}(z, m_Q)$$

LP: Single Parton  
Fragmentation

$$+ \sum_{[Q\bar{Q}(\kappa)]} d\hat{\sigma}_{A+B \rightarrow [Q\bar{Q}(\kappa)]+X}(p(1 \pm \zeta_1)/2z, p(1 \pm \zeta_2)/2z) \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta_1, \zeta_2, m_Q)$$

NLP:  $Q\bar{Q}$   
Fragmentation

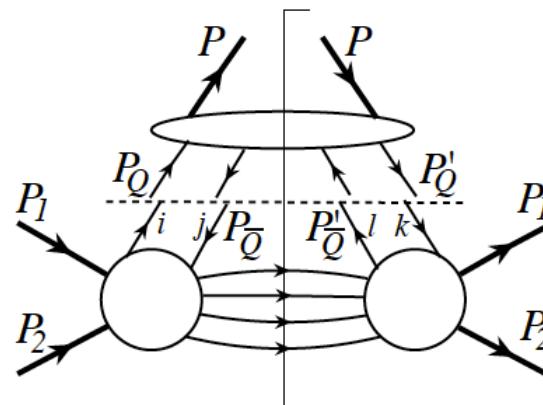
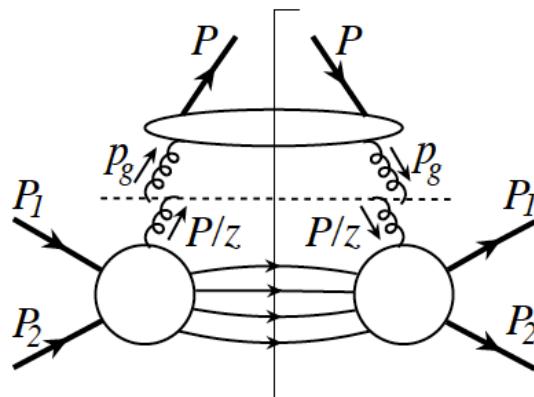
$$+ \mathcal{O}(m_Q^4/p_T^8)$$

3 variables:  $z, \zeta_1, \zeta_2$

Spinor:  
vector (V),  
axial-vector (A),  
tensor (T)  
Color: 1 or 8

Expand cross section first in powers of  $1/p_T$ , then  $\alpha_s$ .

Factorization is proved to all orders in  $\alpha_s$  for both LP and NLP



$\zeta_1 = 1 \Leftrightarrow P_{\bar{Q}} \cdot n = 0$   
 $\zeta_1 = -1 \Leftrightarrow P_Q \cdot n = 0$   
 $\zeta_2$  is for the complex conjugate.  
 $\zeta_1$  and  $\zeta_2$  can be different.

Nayak, Qiu and Sterman, PRD (2005) ... ,  
Kang, Qiu and Sterman, PRL (2011)...  
Kang, Ma, Qiu and Sterman, PRD (2014).  
SCET approach: Fleming et.al., PRD (2012)

# Predictive Power

- ❖ Short-distance hard part can be calculated perturbatively

Choose  $\mu \sim \mathcal{O}(p_T)$ , no large logarithms exist. Perturbation should converge fast.

Hard parts of all channels have been calculated to LO.

Kang, Ma, Qiu, Sterman,  
PRD 91.014030

- ❖ Evolution equations determine the scale dependence of FFs

$$\frac{d}{d \ln \mu^2} D_{H/f}(z, m_Q, \mu) = \sum_j \frac{\alpha_s}{2\pi} \gamma_{f \rightarrow j}(z) \otimes D_{H/j}(z, m_Q, \mu)$$

NLP contributes  
to LP via evolution

$$+ \frac{1}{\mu^2} \sum_{[Q\bar{Q}(\kappa)]} \frac{\alpha_s^2}{(2\pi)^2} \Gamma_{f \rightarrow [Q\bar{Q}(\kappa)]}(z, \zeta, \zeta') \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta_1, \zeta_2, m_Q, \mu)$$

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$$\frac{d}{d \ln \mu^2} \mathcal{D}_{H/[Q\bar{Q}(c)]}(z, \zeta, \zeta', m_Q, \mu) = \sum_{[Q\bar{Q}(\kappa)]} \frac{\alpha_s}{2\pi} K_{[Q\bar{Q}(c)] \rightarrow [Q\bar{Q}(\kappa)]}(z, \zeta, \zeta') \otimes \mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta_1, \zeta_2, m_Q, \mu)$$

By solving the evolution equations,  $\log(\mu^2 / \mu_0^2)$  is resummed.

- ❖ Evolution kernels can be calculated in perturbative QCD

All evolution kernels have been calculated to LO.

Fleming et.al., PRD (2013)  
Kang, Ma, Qiu and Sterman, PRD (2013)

Predictive power of QCD factorization relies on input FFs

# Input fragmentation functions

❖ Input FFs include all non-perturbative interaction

- Relative production rate
- Polarization
- Very hard to be extracted from data

Example: polarization-summed J/ $\psi$

$$D_{H/f}(z, m_Q, \mu_0) \quad f = q, c, b, g \quad 4 \text{ functions}$$

$$\mathcal{D}_{H/[Q\bar{Q}(\kappa)]}(z, \zeta_1, \zeta_2, m_Q, \mu_0) [Q\bar{Q}(\kappa)] = v^{[1,8]}, a^{[1,8]}, t^{[1,8]} \quad 6 \text{ functions}$$

**3 variables**

**20 functions if with polarization**

❖ Different from fragmentation to pion or kaon

- Large heavy quark mass  $m_Q \gg \Lambda_{QCD}$   $\rightarrow$  Partially perturbative
- Separated energy scales in heavy quarkonium  $\rightarrow$  Apply NRQCD to calculate the FFs

**Ten or twenty** unknown functions



**Three** unknown LDMEs

LP fragmentation function: Braaten, Yuan... since 1993

Greatly enhance the predictive power of QCD factorization.

# Matching Input FFs to NRQCD

## ❖ Factorization form in NRQCD factorization

$$D_{f \rightarrow H}(z; m_Q, \mu_0) = \sum_{[Q\bar{Q}(n)]} \hat{d}_{f \rightarrow [Q\bar{Q}(n)]}(z; m_Q, \mu_0, \mu_\Lambda) \langle \mathcal{O}_{[Q\bar{Q}(n)]}^H(\mu_\Lambda) \rangle$$

- Short-distance coefficients  $\hat{d}_{f \rightarrow [Q\bar{Q}(n)]}$  are perturbative, insensitive to the long-distance hadronization process

$$\underline{D_{f \rightarrow [Q\bar{Q}(n')]}}(z; m_Q, \mu_0) = \sum_{[Q\bar{Q}(n)]} \hat{d}_{f \rightarrow [Q\bar{Q}(n)]}(z; m_Q, \mu_0, \mu_\Lambda) \underline{\langle \mathcal{O}_{[Q\bar{Q}(n)]}^{[Q\bar{Q}(n')]}(\mu_\Lambda) \rangle}$$

Calculated with pQCD                              Calculated with NRQCD

↑

Get the coefficient by matching LHS and RHS

- Heavy quark pair FFs are similar.

## ❖ Use conventional dimensional regularization to regulate both UV and IR Divergences

- The NLO short-distance coefficients are finite for all S-wave and P-wave channels.

Support NRQCD factorization

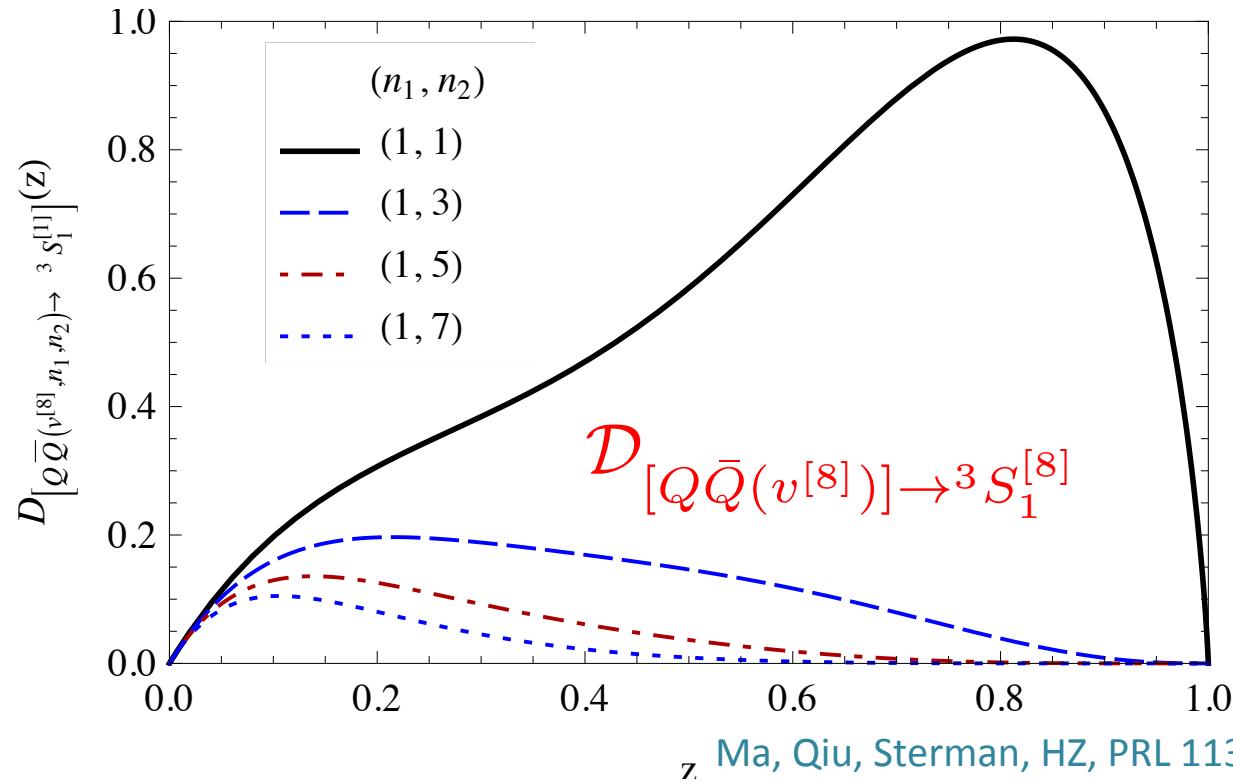
# Input NLP FF: Moments

Unlike the FFs of light hadrons extracted from the data, quarkonium FFs calculated in NRQCD are distributions defined under integration ( $\delta$ , +/-, ...)

Moments:  $\mathcal{D}^{[n_1, n_2]}(z) \equiv \int_{-1}^1 \frac{d\zeta_1 d\zeta_2}{4} \zeta_1^{n_1} \zeta_2^{n_2} \mathcal{D}(z, \zeta_1, \zeta_2)$

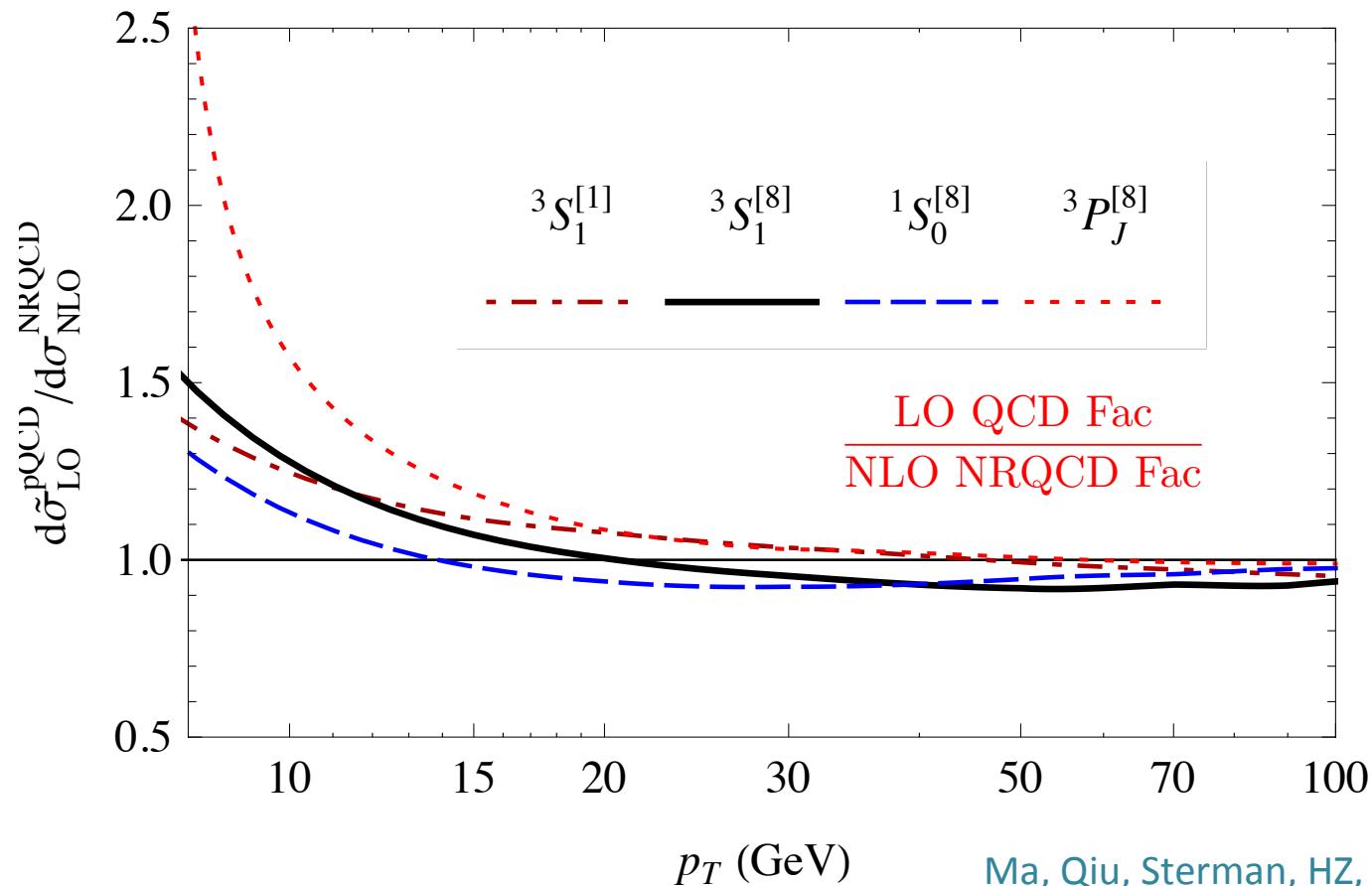
Fragmentation contribution dominated by large z region – two steep falling PDFs

Higher moments decrease quickly for moderate and large z, indicating the probability of a relativistic heavy quark pair with large relative momentum to form quarkonium is small.



# Compare with NLO NRQCD

- ❖ Inclusive hadron production for polarization-summed J/ $\psi$
- ❖ Analytical LO QCD factorization can reproduce the complicated, numerical NLO NRQCD calculation channel by channel at  $p_T > 15$  GeV
- ❖ QCD factorization approach has better control of high-order corrections



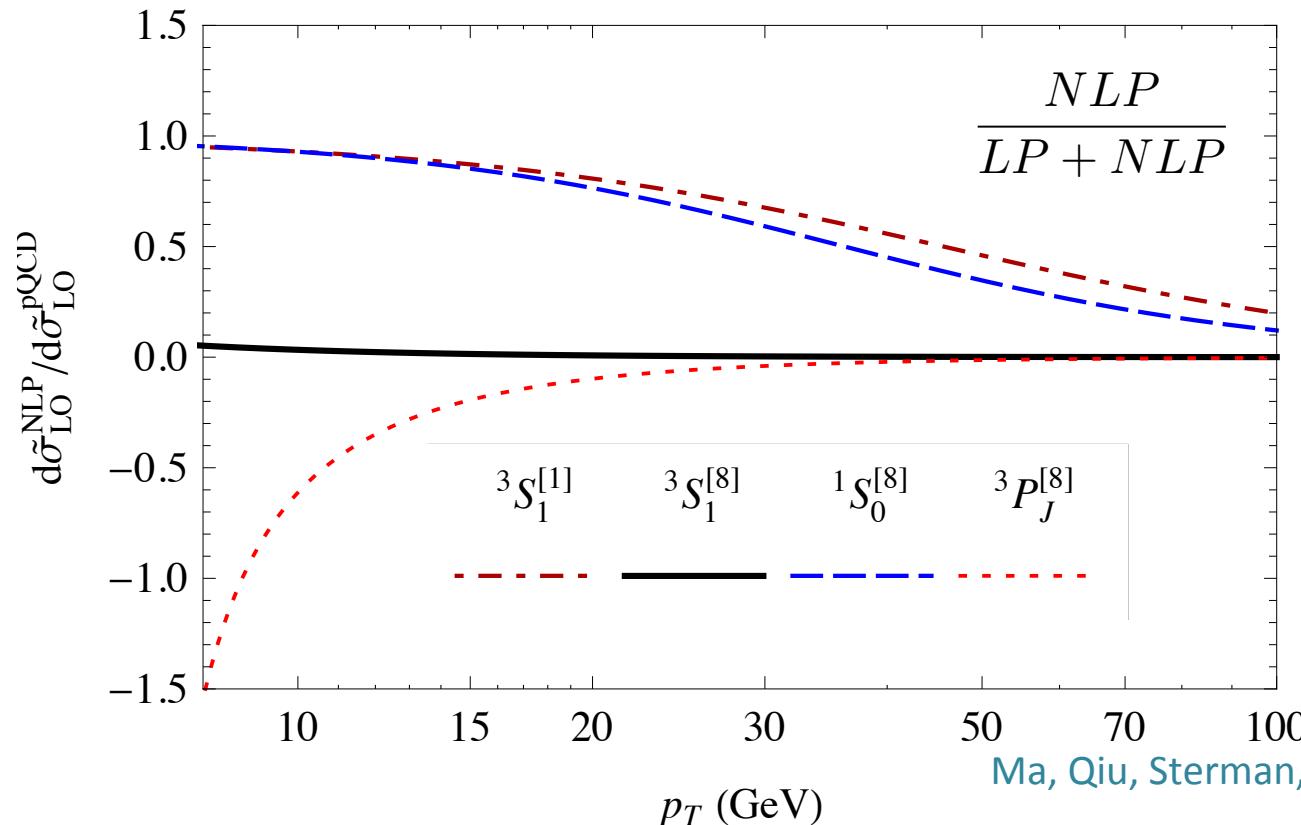
# Importance of NLP

- ❖ NLP is very important at current collider energies

NLP  $^3S_1^{[8]}$  : Negligible

NLP  $^3P_J^{[8]}$  : small when  $p_T > 20$  GeV

NLP  $^3S_1^{[1]}$  and  $^1S_0^{[8]}$  : crucial even at  $p_T \sim 70$  GeV



Ma, Qiu, Sterman, HZ, PRL 113.042002

Use NNLO result for LP fragmentation in  $^3S_1^{[1]}$  channel

Braaten and Yuan, PRD 1993  
13/18  
Bodwin, Kim and Lee, JHEP 2012

# Polarized NRQCD LDMEs

❖ Polarized FF are necessary to solve J/psi polarization puzzle.

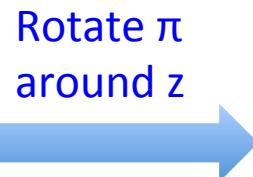
❖ For  ${}^3S_1$  channel      Ma, Qiu, HZ, arXiv: 1501.04556, accepted in JHEP

d-dimensional heavy quark pair in  ${}^3S_1$  state:       $[{}^3S_1] = \chi^\dagger \sigma^i \psi \quad i = 1, 2, \dots, N=d-1$

3-dimensional spherical harmonics

Longitudinal     $Y_{1,0} \propto \cos \theta$

Transverse     $Y_{1,\pm 1} \propto \sin \theta e^{\pm i\phi}$



$$Y_{1,0}$$

$$-Y_{1,\pm 1}$$

This rotation operation can be generalized to d dimensions

$$\text{diag}\{-1, -1, 1\} \longrightarrow \text{diag}\{-1, -1, \dots, 1\}$$

# Polarized NRQCD LDMEs

## ❖ For ${}^3P_J$ channel

d-dimensional heavy quark pair in  ${}^3P_J$  state:  $[{}^3P_J] = \chi^\dagger (-\frac{i}{2} \overleftrightarrow{D}^j) \sigma^k \psi$   $j, k = 1, 2 \dots N-1$

From d=4 L-S coupling

J=0: SO(N) symmetry

J=1: anti-symmetric of L and S

J=2: symmetric of L and S

## ❖ For ${}^3P_2$ channel

$$Y_{2,\pm 2} \propto \sin^2 \theta e^{\pm 2i\phi}$$

$$Y_{2,\pm 1} \propto \sin \theta \cos \theta e^{\pm i\phi}$$

$$Y_{2,0} \propto (3 \cos^2 \theta - 1)$$

Rotating  $\pi$  around z-axis can separate out  $Y_{2,\pm 1}$

$Y_{2,0}$  has SO(N-1) symmetry, with which we separate  $Y_{2,0}$  and  $Y_{2,\pm 2}$

# Polarized NRQCD LDMEs

Ma, Qiu, HZ, arXiv: 1501.04556, accepted in JHEP

$$\mathcal{O}^{H_\lambda}(^3S_{1,T}^{[8]}) = \frac{1}{(d-2)} \chi^\dagger \sigma^{j_\perp} T^a \psi(a_{H_\lambda}^\dagger a_{H_\lambda}) \psi^\dagger \sigma^{j_\perp} T^a \chi, \quad (\text{A.1a})$$

$$\mathcal{O}^{H_\lambda}(^3S_{1,L}^{[8]}) = \chi^\dagger \sigma^z T^a \psi(a_{H_\lambda}^\dagger a_{H_\lambda}) \psi^\dagger \sigma^z T^a \chi, \quad (\text{A.1b})$$

$$\mathcal{O}^{H_\lambda}(^1P_{1,T}^{[8]}) = \frac{1}{(d-2)} \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^{j_\perp}\right) T^a \psi(a_{H_\lambda}^\dagger a_{H_\lambda}) \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^{j_\perp}\right) T^a \chi, \quad (\text{A.1c})$$

$$\mathcal{O}^{H_\lambda}(^1P_{1,L}^{[8]}) = \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^z\right) T^a \psi(a_{H_\lambda}^\dagger a_{H_\lambda}) \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^z\right) T^a \chi, \quad (\text{A.1d})$$

$$\mathcal{O}^{H_\lambda}(^3P_{1,T}^{[8]}) = \frac{1}{2(d-2)} \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^{[j_\perp \sigma^z]}\right) T^a \psi(a_{H_\lambda}^\dagger a_{H_\lambda}) \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^{[j_\perp \sigma^z]}\right) T^a \chi, \quad (\text{A.1e})$$

$$\mathcal{O}^{H_\lambda}(^3P_{1,L}^{[8]}) = \frac{1}{2(d-2)(d-3)} \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^{[j_\perp \sigma^{k_\perp}]}\right) T^a \psi(a_{H_\lambda}^\dagger a_{H_\lambda}) \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^{[j_\perp \sigma^{k_\perp}]}\right) T^a \chi, \quad (\text{A.1f})$$

$$\begin{aligned} \mathcal{O}^{H_\lambda}(^3P_{2,TT}^{[8]}) &= \frac{2}{(d-1)(d-2)-2} \chi^\dagger \left(-\frac{i}{2} \left(\frac{1}{2} \overleftrightarrow{D}^{\{j_\perp \sigma^{k_\perp}\}} - \frac{\delta^{j_\perp k_\perp}}{d-2} \overleftrightarrow{\mathbf{D}}_T \cdot \boldsymbol{\sigma}_T\right)\right) T^a \psi \\ &\quad (a_{H_\lambda}^\dagger a_{H_\lambda}) \psi^\dagger \left(-\frac{i}{2} \left(\frac{1}{2} \overleftrightarrow{D}^{\{j_\perp \sigma^{k_\perp}\}} - \frac{\delta^{j_\perp k_\perp}}{d-2} \overleftrightarrow{\mathbf{D}}_T \cdot \boldsymbol{\sigma}_T\right)\right) T^a \chi, \end{aligned} \quad (\text{A.1g})$$

$$\mathcal{O}^{H_\lambda}(^3P_{2,T}^{[8]}) = \frac{1}{2(d-2)} \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^{\{j_\perp \sigma^z\}}\right) T^a \psi(a_{H_\lambda}^\dagger a_{H_\lambda}) \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{D}^{\{j_\perp \sigma^z\}}\right) T^a \chi, \quad (\text{A.1h})$$

$$\begin{aligned} \mathcal{O}^{H_\lambda}(^3P_{2,L}^{[8]}) &= \frac{d-2}{d-1} \chi^\dagger \left(-\frac{i}{2} (\overleftrightarrow{D}^z \sigma^z - \frac{1}{d-2} \overleftrightarrow{\mathbf{D}}_T \cdot \boldsymbol{\sigma}_T)\right) T^a \psi \\ &\quad (a_{H_\lambda}^\dagger a_{H_\lambda}) \psi^\dagger \left(-\frac{i}{2} (\overleftrightarrow{D}^z \sigma^z - \frac{1}{d-2} \overleftrightarrow{\mathbf{D}}_T \cdot \boldsymbol{\sigma}_T)\right) T^a \chi, \end{aligned} \quad (\text{A.1i})$$

$$\mathcal{O}^{H_\lambda}(^3P_0^{[8]}) = \frac{1}{d-1} \chi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma}\right) T^a \psi(a_{H_\lambda}^\dagger a_{H_\lambda}) \psi^\dagger \left(-\frac{i}{2} \overleftrightarrow{\mathbf{D}} \cdot \boldsymbol{\sigma}\right) T^a \chi, \quad (\text{A.1j})$$

# NLP FF is mainly longitudinal

- ❖ All short-distance coefficients of polarized FFs are **finite** in NLO NRQCD calculation.

Ma, Qiu, HZ, arXiv: 1501.04556

- ✧ LP: contribute to transversely polarized J/ $\psi$ .
- ✧ NLP: contribute mainly to longitudinally polarized J/ $\psi$ .

LO results of important channels

|           | $^3S_1^{[1]}$ | $^3S_1^{[8]}$ | $^3P_J^{[8]}$ | $^1S_0^{[8]}$ |
|-----------|---------------|---------------|---------------|---------------|
| g         |               | T             |               |               |
| $v^{[1]}$ | L             |               |               |               |
| $v^{[8]}$ |               | L             | L             |               |
| $a^{[1]}$ |               |               |               |               |
| $a^{[8]}$ |               |               | T             | Un            |

- ❖ In addition to direct contribution, NLP FFs at input scale  $\mu_0 \gtrsim 2m_Q$  also contribute to LP FFs at larger scale via the mixed kernel in the evolution

$$\frac{d}{d \ln \mu^2} D_{H/f}(z, m_Q, \mu) = \sum_j \frac{\alpha_s}{2\pi} \gamma_{f \rightarrow j} \otimes D_{H/j} + \frac{1}{\mu^2} \sum_{[Q\bar{Q}(\kappa)]} \frac{\alpha_s^2}{(2\pi)^2} \Gamma_{f \rightarrow [Q\bar{Q}(\kappa)]}(z, \zeta_1, \zeta_2) \otimes \mathcal{D}_{H/[Q\bar{Q}](\kappa)}(z, \zeta_1, \zeta_2, m_Q, \mu)$$

- ❖ At moderate  $p_T$ , the LP and NLP contribution compete and result in unpolarized J/ $\psi$ .

With the calculated polarized FFs, NLP QCD factorization is very promising to solve the long-standing polarization puzzle.

# Summary and outlook

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- ❖ NLP QCD factorization proposes expansion of  $1/p_T$  before  $\alpha_s$ , and proved the factorization to all orders of  $\alpha_s$  for both LP and NLP.
- ❖ Assuming NRQCD factorization at input scale  $\mu_0 \gtrsim 2m_Q$ , we calculated these input unpolarized and polarized quarkonium FFs up to a few NRQCD LDMEs.
- ❖ LO hard parts  $\otimes$  LO FFs reproduce “very complicated” NLO NRQCD calculation, clearly showing the  $1/p_T$  expansion of cross section is better organized.
- ❖ With polarized FF calculated, QCD factorization is very promising to solve the polarization puzzle.
- ❖ For universality puzzle, global analysis needs more data from various processes, and more calculations.

See also talks by Bodwin and Qiu

Thank you!

# Backup slides

# Special Divergences Cancellation for FFs

- ❖ UV: Need renormalization for composite operators, in addition to QCD renormalization
- ❖ IR: due to the parameters  $z, \zeta_1, \zeta_2$

- Single Parton FF: cancellation between virtual & real

Virtual diagrams:  $\frac{1}{\epsilon_{IR}} \delta(1-z)$       Real diagrams:  $(1-z)^{-1-2\epsilon}$       Diverge at  $z \rightarrow 1$

$$(1-z)^{-1-2\epsilon} = -\frac{1}{2\epsilon_{IR}} \delta(1-z) + \frac{1}{(1-z)_+} - 2\epsilon \left( \frac{\log(1-z)}{(1-z)} \right)_+$$

$$\int_0^1 dz \frac{1}{(1-z)_+} f(z) = \int_0^1 dz \frac{f(z) - f(1)}{1-z} \quad \text{Converge at } z \rightarrow 1$$

- Heavy quark pair FF: additional amplitude level cancellation

$\frac{1}{\epsilon_{IR}} \delta(\zeta_1)$  and  $\frac{1}{\epsilon_{IR}} \delta'(\zeta_1)$        $\zeta_1^{-1-2\epsilon}$  and  $\zeta_1^{-2-2\epsilon}$   
Diverge at  $\zeta_1 \rightarrow 0$

Generalized plus/minus-distributions

$$\int d\zeta_1 \left[ \frac{1}{\zeta_1^n} \right]_{n\pm} f(\zeta_1) = \int_{-1}^1 d\zeta_1 \left( \frac{\theta(\zeta_1)}{\zeta_1^n} \pm \frac{\theta(-\zeta_1)}{(-\zeta_1)^n} \right) \left[ f(\zeta_1) - \sum_{i=0}^{n-1} \frac{1}{i!} f^{(i)}(0) \zeta_1^i \right]$$

Similarly for  $\zeta_2$  in the complex conjugate of the amplitude.

# Example: NLP FFs

Example:  $[Q\bar{Q}(a^{[8]}) \rightarrow [Q\bar{Q}(^1S_0^{[8]})]]$  Both virtual and real contributions

✧ Leading order:

$$\hat{d}_{[Q\bar{Q}(a^{[8]})] \rightarrow [Q\bar{Q}(^1S_0^{[8]})]}^{\text{LO}}(z, \zeta_1, \zeta_2; m_Q, \mu_0) = \frac{1}{N_c^2 - 1} \frac{1}{2m_Q} \delta(1 - z) \delta(\zeta_1) \delta(\zeta_2)$$

✧ Next-to-leading order:

$$\begin{aligned} \hat{d}_{[Q\bar{Q}(a^{[8]})] \rightarrow [Q\bar{Q}(^1S_0^{[8]})]}^{\text{NLO}}(z, \zeta_1, \zeta_2, \mu_0; m_Q) &= \frac{\alpha_s}{64 \pi m_Q (N_c^2 - 1)} \quad \text{Heavy quark pair evolution kernel} \\ &\times \left\{ \Gamma_{[Q\bar{Q}(a^{[8]})] \rightarrow [Q\bar{Q}(a^{[8]})]}(z, \frac{1 + \zeta_1}{2}, \frac{1 + \zeta_2}{2}; \frac{1}{2}, \frac{1}{2}) \ln \left[ \frac{\mu_0^2}{m_Q^2} \right] \right. \\ &\quad \left. + R(z, \zeta_1, \zeta_2) + \delta(1 - z)[V(\zeta_1)\delta(\zeta_2) + V(\zeta_2)\delta(\zeta_1)] \right\} \end{aligned}$$

$$V(\zeta_1) = \frac{1}{N_c} \left\{ 2 \left[ - \left( \frac{1}{\zeta_1^2} \right)_{2+} + \left( \frac{1}{\zeta_1} \right)_{1+} + \left( \frac{\ln(\zeta_1^2)}{\zeta_1} \right)_{1+} \right] - [(\zeta_1)_{0+} + 1] \ln(\zeta_1^2) - (\zeta_1)_{0+} + 1 \right\}$$

Remnants of amplitude level  
IR divergence cancellation

$$R(z, \zeta_1, \zeta_2) = \frac{1}{N_c} \left\{ \Delta_+^{[8]} \left[ -2z \left( \frac{\ln(2 - 2z)}{1 - z} \right)_+ - \frac{z}{(1 - z)_+} \right] - 8 [(\ln 2)^2 + \ln 2] C_A^2 \delta(\zeta_1) \delta(\zeta_2) \delta(1 - z) \right\}$$

Remnants of IR pole cancellation between real and virtual corrections

# NLP contribution is important

Power expansion of the cross section

$$\frac{d\sigma_{pp \rightarrow J/\psi}}{dp_T^2} = \hat{\sigma}_{pp \rightarrow a}^{\text{LP}} \otimes D_{a \rightarrow J/\psi} + \hat{\sigma}_{pp \rightarrow Q\bar{Q}}^{\text{NLP}} \otimes \mathcal{D}_{Q\bar{Q} \rightarrow J/\psi} + \mathcal{O}\left(\frac{1}{p_T^8}\right)$$

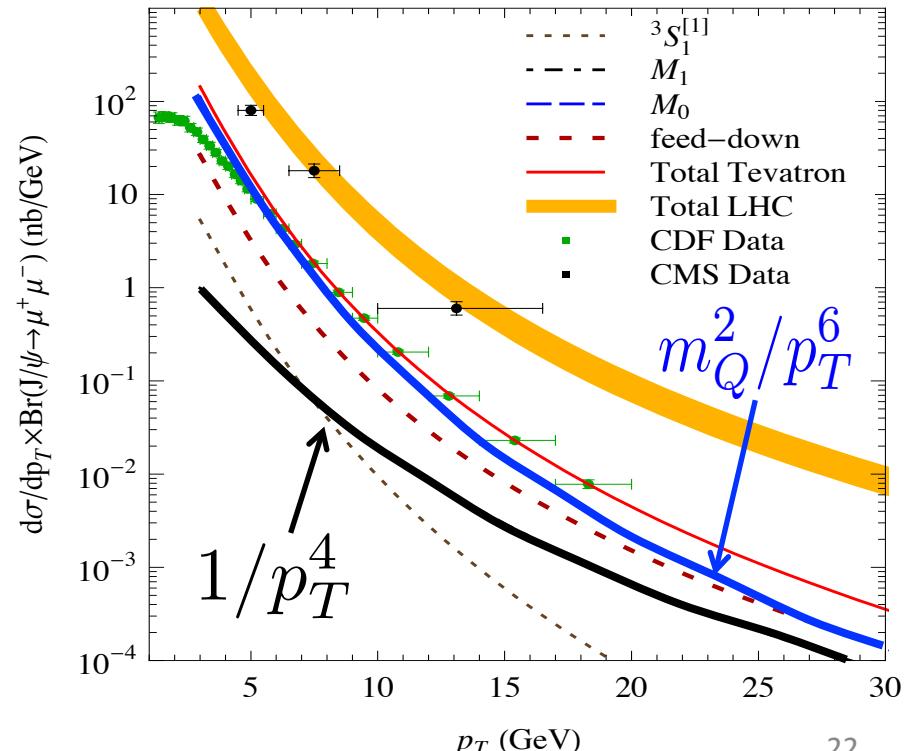
$\mathcal{O}(1/p_T^4)$

$\mathcal{O}(m_Q^2/p_T^6)$

Intuitively, easier to find a quarkonium in a heavy quark pair than in a single parton.

## ➤ Importance of NLP

- Direct evidence:  
 $1/p_T^6$  is shown large in the unpolarized  $J/\psi$  production in current collider energies.
- Indirect evidence:  
 $J/\psi$  and  $\Upsilon$  are produced almost unpolarized.



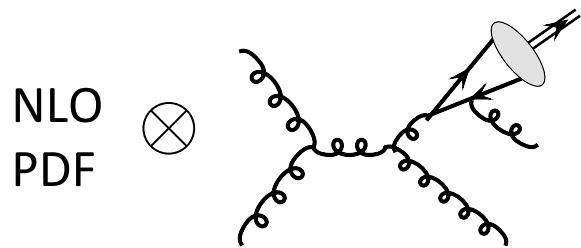
Ma, Wang, Chao PRL (2011)

# Compare with NLO NRQCD

| Channel Power | $^3S_1^{[1]}$<br>LP | $^3S_1^{[1]}$<br>NLP | $^3S_1^{[8]}$<br>LP | $^3S_1^{[8]}$<br>NLP | $^1S_0^{[8]}$<br>LP | $^1S_0^{[8]}$<br>NLP | $^3P_J^{[8]}$<br>LP | $^3P_J^{[8]}$<br>NLP |
|---------------|---------------------|----------------------|---------------------|----------------------|---------------------|----------------------|---------------------|----------------------|
| PDFs          | ...                 | LO                   | LO                  | LO                   | NLO                 | LO                   | NLO                 | LO                   |
| FFs           | ...                 | $\alpha_s^1$         | $\alpha_s^1$        | $\alpha_s^0$         | $\alpha_s^2$        | $\alpha_s^0$         | $\alpha_s^2$        | $\alpha_s^0$         |
| SDCs          | ...                 | $\alpha_s^3$         | $\alpha_s^2$        | $\alpha_s^3$         | $\alpha_s^2$        | $\alpha_s^3$         | $\alpha_s^2$        | $\alpha_s^3$         |

Example:  $^1S_0^{[8]}$  channel

➤ NLO NRQCD factorization



➤ LO QCD factorization

